Exam 2 Review

Warm Up: Find these derivatives.

Ex: $f(x) = 3x^4 + 6x^2 + 5x + 1$

$f'(x) = 12x^3 + 12x + 5$

Ex: $g(x) = x^2 e^{3x+1}$ *product rule

$g'(x) = (2x)e^{3x+1} + x^2 e^{3x+1} \cdot 3$

$= 2x e^{3x+1} + 3x^2 e^{3x+1}$

$= (2x + 3x^2) e^{3x+1}$

Ex: $h(x) = \dfrac{x+1}{3x+2}$

* quotient rule

$$h'(x) = \frac{(1)(3x+2) - (x+1)(3)}{(3x+2)^2}$$

$$= \frac{3x+2-3x-3}{(3x+2)^2}$$

$$= \frac{-1}{(3x+2)^2}$$

Ex: let $f(x) = 2x^4 + x^3 - 5x^2 + 3$

find $f''(x)$.

$$f'(x) = 8x^3 + 3x^2 - 10x$$

$$f''(x) = 24x^2 + 6x - 10$$

Ex: I save $100 and invest it at a rate of 3% compounded continuously. How long will it take for my investment to quadruple?

$$P = P_0 e^{rt}$$

$$400 = 100 e^{.03t}$$

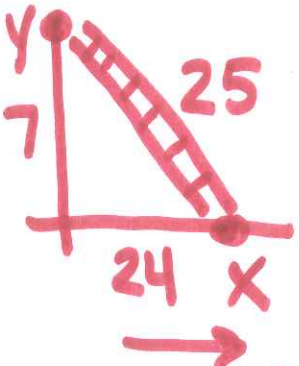
$$4 = e^{.03t}$$

$$\ln 4 = \ln e^{.03t}$$

$$\ln 4 = .03t$$

$$t = \frac{\ln 4}{.03} \approx 46.2 \text{ years}$$

Ex: a 25 ft ladder is leaning against a wall. If the bottom slides away at a rate of 2 ft/sec, how fast is the top sliding down when the bottom is 24 ft from the wall?



when $x = 24$ find y

$$24^2 + y^2 = 25^2$$

then $y = 7$

$x^2 + y^2 = 25^2$ take derivative with respect to t!

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(24)(2) + 2(7)\frac{dy}{dt} = 0$$

$$96 + 14\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-96}{14} \approx -6.86 \text{ ft/sec}$$

6.86 ft/sec

Ex: let $f(x) = x^3 + 5x^2 + 3x + 4$. Find the max and min on the interval $[-5, 5]$.

$f(x)$ defined, continuous, differentiable on $[-5,5]$ ✓

$f'(x) = 3x^2 + 10x + 3$ defined on $[-5,5]$ ✓

$$3x^2 + 10x + 3 = 0$$
$$(3x+1)(x+3) = 0$$
$$x = -\frac{1}{3}, -3$$

Check:

$$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 + 5\left(-\frac{1}{3}\right)^2 + 3\left(-\frac{1}{3}\right) + 4 = \frac{65}{27} \approx 2.4$$
$$f(-3) = (-3)^3 + 5(-3)^2 + 3(-3) + 4 = 13$$
$$f(-5) = (-5)^3 + 5(-5)^2 + 3(-5) + 4 = -11$$
$$f(5) = (5)^3 + 5(5) + 3(5) + 4 = 269$$

max at $(5, 269)$
min at $(-5, -11)$